Multiscale coupling in ocean and climate modeling

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Motivation

- Turbulence parameterization in Geophysical Fluid Dynamics

- Requirements of next generation of ocean/climate models
North-Atlantic simulation 2003

- Parallel Ocean Program, 1/10° resolution, hydrostatic, Rossby deformation radius resolved ($L_R \sim 10\text{-}50\text{km}$)

- Next step: Non-hydrostatic effects → even smaller eddies
Multiscale coupling in the ocean

Interscale coupling of slow, large scale coherent motions: great ocean conveyor belt and fast, three-dimensional small-scale mixing: dense water overflow.

Theoretical understanding, diagnostic tools and parametrizations for the next generation of ocean, atmosphere and climate models.
Next generation of models

- Non-hydrostatic small-scale effects are currently ignored, but they have $O(1)$ effects over long times.
- Small scale (sub-deformation scale, non-hydrostatic) effects must be explicitly calculated, modeled or parameterized.
- What are the effects of the small scales on the large?
Multiscale coupling in ocean and climate modeling

People

- Susan Kurien, Beth Wingate, Nicole Jeffery (postdoc) (Los Alamos National Laboratory)
- Prof. Leslie Smith, Prof. Zhengyu Liu, Jai Sukhatme (postdoc), Mark Remmel (student), Li Wang (student) (University of Wisconsin, Madison)
- Mark Taylor (Sandia National Laboratories)
- Summer students at LANL: Miranda Holmes (Courant Institute of Mathematical Sciences), Mike Watson (University of Colorado, Boulder)
Science cornerstones

Connect turbulence and GFD.

- classical turbulence theory does not account for multiple spatial-scale and time-scale dependent parameters (Reynolds (Re), rotation (Ro), stratification (Fr)).
- adapt mathematical tools from turbulence theory to capture the multiscale, multi-parameter nature of GFD.

Scale-linking due to nonhydrostatic effects.

- small-scale vertical mixing and sub-deformation scale effects need to be accounted for in ocean and climate modeling.
Connecting turbulence and GFD
Key results

- new statistical benchmark for rotating/stratified turbulence (Kurien, Smith & Wingate, J. Fluid Mech., 2006)
- new constraints on energy transfer due to potential enstrophy in strongly rotating/stratified turbulence (Kurien, Wingate, Taylor, to be submitted, 2007)
- ongoing verification of new diagnostics using high-performance DNS code
Non-rotating, non-stratified turbulence

- benchmarks (Kolmogorov 1941)

scale-by-scale energy flux:

\[ \Delta u_l = (u(x + r) - u(x)) \cdot \hat{r} \]

\[ \left\langle (\Delta u_l(r))^3 \right\rangle = -\frac{4}{5} \varepsilon r \rightarrow E(k) = C \varepsilon^{2/3} k^{-5/3} \]

- Reynolds number \( Re = UL/\nu \) is the only parameter.
- \( Re \rightarrow \infty \) is the only sensible limit.
K41 theory has implications for turbulence modeling

- **benchmark** for calculations, models and theory.
- **physics** of the modeled scales can be characterized by K41 statistical parameters
  - eg. Smagorinsky model constant assumes $k^{-5/3}$ scaling of energy spectrum.
Rotating and stratified turbulence has wider parameter space

- Strongly rotating
- Strongly stratified
- 3D turbulence
- Laminar

\( \text{Rossby } \text{Ro} = \frac{U}{Lf} \)
\( \text{Froude } \text{Fr} = \frac{U}{LN} \)
\( \text{Reynolds } \text{Re} = \frac{UL}{\nu} \)
Beyond energy -- dependence on \( Ro \) and \( Fr \)

- **potential vorticity** locally conserved
  \[
  q = \omega_a \cdot \nabla \rho
  \]
  \[
  \omega_a = \omega + 2\Omega
  \]
  \[
  \rho = \rho_0 + b z + \tilde{\rho}
  \]

  non-dimensional form, \( Ro \) and \( Fr \) dependence

  \[
  q = \omega \cdot \nabla \tilde{\rho} + Ro^{-1} \frac{\partial \tilde{\rho}}{\partial z} - Fr^{-1} \omega_3
  \]

- **potential enstrophy** conserved

  \[
  Q = q^2
  \]
Potential enstrophy and energy cascades: **Quasi-geostrophy**

- **Approximation** for large-scale rotating and stratified flow, assumes small scales are dynamically unimportant.

- Charney (1971): Potential enstrophy conservation suppresses forward cascade of energy, and scaling of energy spectrum in the high wave numbers:

\[
E(k) = \frac{1}{2} \sum_{|k'|=|k|} |\tilde{u}(k')|^2 \sim k^{-3}
\]

\[
P(k) = \frac{1}{2} \sum_{|k'|=|k|} |\tilde{\theta}(k')|^2 \sim k^{-3}
\]
Potential enstrophy and energy away from quasi-geostrophy

- Our new results begin to include small-scale effects in rotating and stratified flows
  - law for flux of potential enstrophy,
    \[
    \langle qq' (u_l - u_i) \rangle = \frac{2}{3} \varepsilon_Q r
    \]  
    (Kurien, Smith & Wingate (2006))

- scaling laws for potential energy and horizontal kinetic energy spectra,
  (Kurien, Wingate & Taylor (2007))
  
  \[
  \text{Case } \frac{k_h}{k_z} \ll 1 \quad P(k_h, k_z) \sim \varepsilon_Q^{2/5} k_z^{-3}
  \]
  \[
  \text{Case } \frac{k_z}{k_h} \ll 1 \quad E_h(k_h, k_z) \sim \varepsilon_Q^{2/5} k_h^{-3}
  \]
Statistical law for potential enstrophy flux $\varepsilon_Q$

- six different limits in $Ro$ and $Fr$
- new `2/3-law' law for potential enstrophy flux

\[ \langle qq' (u'_l - u_l) \rangle = -\frac{2}{3} \varepsilon_Q r \]

analogous to K41 `4/5-law' for kinetic energy flux
LANL high-performance DNS code

$256^3$ section of $2048^3$ simulation of decaying turbulence on ASC-Q
(Mark Taylor, 2003)

More on data and diagnostic techniques:
Numerical simulations of rotating and stratified turbulence

- periodic box, uniform grid, 512 gridpoints per side
- rotating and stratified in z-direction
- unit aspect ratio
- Boussinesq equations: $\rho << \rho_0$
- stochastically forced at wavenumber $k_f = 4$
- tunable $Ro, Fr, Pr, Re$ and aspect ratio
- results from $(Ro, Fr) \sim 0.001$ (very rapidly rotating and stably stratified)
Scalar field at various heights

- vertically coherent structures emerge
Charney scaling not observed, flow is not QG since small-scale waves are retained.
potential enstrophy suppresses potential energy in the nearly vertical modes

Case \[ \frac{k_h}{k_z} \ll 1 \quad P(k_h, k_z) \sim \varepsilon_{Q}^{2/5} k_z^{-3} \]
potential enstrophy suppresses horizontal kinetic energy in the nearly horizontal modes

\[ C \text{ase } \frac{k_z}{k_h} \ll 1 \quad E_h(k_h, k_z) \sim \varepsilon_Q^{2/5} k_h^{-3}. \]
Back to original motivation

- Turbulence parameterization in GFD
  - benchmark law for potential enstrophy flux: new “2/3-law” in a wide parameter space (Rossby, Froude values).
  - characterization of the small-scales, away from classical QG: derived predictable scaling exponents for energy due to constraining effect of potential enstrophy.

- New results anticipate the next generation of models